Towards a loophole-free test of Bell's inequality with entangled pairs of neutral atoms

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Experimental tests of Bell's inequality allow to distinguish quantum mechanics from local hidden variable theories. Such tests are performed by measuring correlations of two entangled particles (e.g. polarization of photons or spins of atoms). In order to constitute conclusive evidence, two conditions have to be satisfied. First, strict separation of the measurement events in the sense of special relativity is required ("locality loophole"). Second, almost all entangled pairs have to be detected (for particles in a maximally entangled state the required detector efficiency is $2(\sqrt{2}-1) \approx 82.8\%$), which is hard to achieve experimentally ("detection loophole"). By using the recently demonstrated entanglement between single trapped atoms and single photons it becomes possible to entangle two atoms at a large distance via entanglement swapping. Combining the high detection efficiency achieved with atoms with the space-like separation of the atomic state detection events, both loopholes can be closed within the same experiment. In this paper we present estimations based on current experimental achievements which show that such an experiment is feasible in future.

I. INTRODUCTION

In 1935 Einstein, Podolsky and Rosen (EPR) asked the seemingly innocent question, whether quantum mechanics can be considered complete. If not, this might be cured by additional parameters of a physical system (now called local hidden variables, LHV) which are not - yet - known to us. Later, Bell showed, that experimental tests can be performed which allow to decide whether the concept of LHV indeed can be used to describe nature. This proposal triggered a series of experiments, most importantly by Freedman & Clauser[1] and by the group of Alain Aspect[2, 3]. More recently, new experimental techniques enabled Bell-tests with photon pairs from parametric down-conversion and, with the realm of quantum logic, for trapped ions, nuclear spins etc.

So far, all experiments to test Bell's inequalities required additional assumptions, thus opening loopholes in Bell's original argument[4]. The first is called the locality loophole, in which the correlations of apparently separate events could result from unknown subluminal signals influencing the measurement results during the observation of an entangled pair[5, 6]. One experiment was performed with entangled photons[7] enforcing strict relativistic separation between the measurements. But it suffered from low detection efficiencies. It thus opens the second loophole by allowing the possibility that the subensemble of detected events agrees with quantum mechanics even though the entire ensemble satisfies the limits for local-realistic theories as given by Bell's inequalities[8, 9]. This is also referred to as detection loophole and was addressed in an experiment with two trapped ions[10], where the quantum state detection was performed with almost perfect efficiency. But there the ion separation was too small to eliminate the locality loophole.

Based on the experiments performed in our group[11, 12], a final test of LHV-theories[13] comes into reach of our experimental techniques. For this purpose two photons, each entangled with a trapped ^{87}Rb atom, will be distributed far enough to ensure space-like separation, see Fig. 1. A projection of the photons onto Bell-states serves to swap the entanglement to the atoms[14] whose states now can be observed with high efficiency. This enables the ideal configuration of a so called event-ready scheme[4, 5, 14], which does not require any assumptions at all.

II. EXPERIMENTAL REQUIREMENTS

Let us now analyze the experimental requirements. Crucial for such a test is a highly efficient state analysis performed by space-like separated observations on entangled atoms. Here the minimum distance between the atoms is determined by the duration of the atomic state detection process.

The currently used atomic state detection method is a two-step process[11]. It consists of a stimulated Raman adiabatic passage technique (STIRAP) which transfers a selected superposition of the atomic spin states to a different hyperfine level ($F = 2$) and a subsequent detection of the hyperfine state. While the STIRAP process is inherently coherent, the coherence of the atomic state is destroyed right after the STIRAP sequence by resonant scattering of photons within 300 ns with a probability exceeding 99%. Alternatively, the hyperfine state detection can be replaced by state-selective ionization with subsequent detection of the ionization fragments. By irreversibly removing the valence electron, the coherence of the atom is destroyed (according to calculations) after 200 ns with a probability of $> 99\%$. Together with the random choice of the measurement basis (100 ns), the STIRAP process (120 ns), and flight times of the ionization fragments (< 500 ns) it gives an overall detection time of less than 1 μs. The corresponding distance of 300 m between the atoms for closing the lo-

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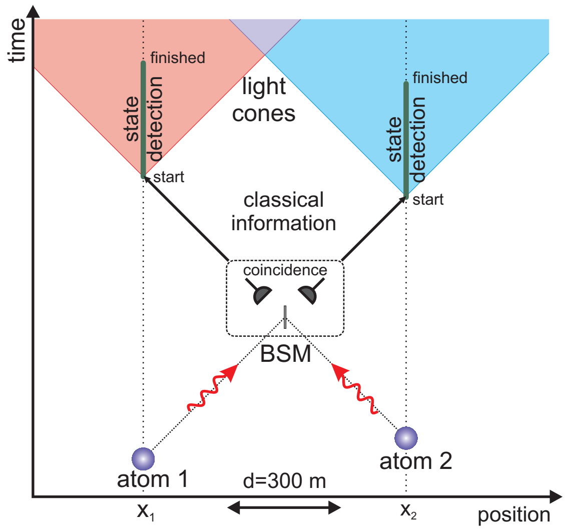

Figure 1: Space-time schematic of the proposed loophole-free Bell experiment. Two atomic traps are separated by 300 m, each atom emits a photon whose polarization is entangled with the atomic spin. The two photons arrive simultaneously on a non-polarizing beamsplitter where interference takes place. The coincidence detection in the outputs of the beamsplitter (equivalent to a Bell-state measurement (BSM) on the two photons) signals the projection of the atoms onto an entangled state. The signal of successful BSM is sent back to both setups, where atomic state detection is started. The detection is performed in a randomly chosen basis and has to be finished before any classical signal can reach the other side (i.e. within less than 1 μs).

cality loophole can easily be achieved since the transmission losses in optical fibers for the photons used for entanglement swapping (wavelength 780 nm) are low (for a demonstration of an optical fiber link of 300 m length see[12]). We emphasize that our scheme is also independent of any detection related loopholes, because entanglement swapping enables the event ready scheme[4, 5, 14], where binary measurement results are reported for every run, started after a joint photon detection event in the Bell-state measurement. For limited detection efficiency/accuracy, however, the obtained results are not always correct. This leads to a reduction of the expected spin correlations. The corresponding accuracies of the two detection methods are analyzed in this paper and the expected violation of Bell's inequality is given.

A. State-selective atom removal

The currently used detection of the hyperfine state involves state-selective removal of the atoms from the trap, which is verified by counting photons collected from the trap region. The mean accuracy of this procedure was experimentally determined to be $a_{HF} = 97.8\%$[15]. Together with the accuracy of the STIRAP process, $a_{ST} = 97.25\%$ it results in an overall detection accuracy of $a_{det}^{(flr.)} = 95\%$. This number specifies the (symmetric) probability for correct identification of the analyzed atomic state (i.e. $|\downarrow\rangle$ is identified as $|\downarrow\rangle$ and $|\uparrow\rangle$ as $|\uparrow\rangle$). A disadvantage of this method due to very low collection efficiency of only about 10^{-3} is the long duration of sampling fluorescence photons until the outcome can be determined (10..20 ms). Yet, one should note that decoherence (coupling to the environment) already takes place within short time (300 ns) by scattering a single photon.

B. State-selective ionization

Alternatively, in order to enable a very fast and direct detection of the atomic state, state-selective ionization can be used. Here again a selected superposition of atomic spin states is first transferred to $5^2S_{3/2}, F = 2$ hyperfine level using the STIRAP technique. Then the atom in $F = 2$ level is optically excited to the $5^2P_{3/2}, F = 3$ level and ionized using an additional laser at a wavelength of 473 nm. The rate of this two-photon ionization process depends on the available intensity of the lasers. We expect to achieve an ionization probability of $p_{ionize} > 99\%$ within 200 ns. The resulting free electron e^- and Rubidium ion $^{87}Rb^+$ can be detected by channel electron multipliers. As it is sufficient to detect at least one of the ionization fragments, the overall detection efficiency p_{det} is given by

$$p_{det} = 1 - (1 - p_e)(1 - p_{ion}). \qquad (1)$$

This method is currently investigated in our group. First calibration measurements for ionization of Rubidium atoms from background gas in a vacuum cell show efficiencies of $p_e = 80\%$ and $p_{ion} = 60\%$. The goal is to reach values $p_e \geq 85\%$ and $p_{ion} \geq 65\%$, which would give a detection efficiency of $p_{det} = 95\%$ and better.

Again it has to be stressed that the efficiency for detection of ionization fragments is not the detection efficiency in the Bell experiment. Due to the binary nature of the result (either a fragment is detected corresponding to the measurement result "$|\uparrow\rangle$", or it is not detected, corresponding to the measurement result "$|\downarrow\rangle$", but a result is always given) this efficiency does only influence the accuracy of the state detection.

III. EXPECTED VISIBILITY FOR THE ENTANGLEMENT SWAPPING

For all further considerations we assume that the entangled state of atom-photon or two atoms has the density matrix of the following form

$$\hat{\rho} = V\,|\Psi\rangle\,\langle\Psi| + (1-V)\frac{1}{4}\hat{1}, \qquad (2)$$

where V is the visibility, $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle\,|\uparrow\rangle \pm |\uparrow\rangle\,|\downarrow\rangle)$ is a maximally entangled state and $\frac{1}{4}\hat{1}$ is the density matrix of the completely mixed state[16]. In a correlation measurement, where the relative angle between the measurement bases of the two particles is varied, the visibility V describes the difference between the maximum and the minimum values (also called contrast) of the observed interference fringe. Given the state represented by the density matrix $\hat{\rho}$ from (2), the probability to

find the two particles in the (pure) state $|\Psi\rangle$ (also called the fidelity F) is $F = \frac{1}{4} + \frac{3}{4}V$.

For any additional error occurring at the further stages of the experiment we assume that the density matrix is modified like

$$\hat{\rho} \rightarrow (1-e)\hat{\rho} + e \cdot \frac{1}{4}\hat{1},$$

where e is the error probability. This assumes that any error results in a completely mixed state. For visibility V and fidelity F of the state follows

$$\begin{aligned} V \rightarrow& (1-e)V, \\ F \rightarrow& (1-e)F + \frac{1}{4}e. \end{aligned} \qquad (3)$$

These relations allow to calculate the influence of different errors during the transmission of the state, entanglement swapping, etc.

In order to generate an entangled pair of atoms, the starting situation is the emission of a photon by the atom. During this process the polarization of the photon gets entangled with the respective atomic spin resulting in the maximally entangled state[11]

$$\left|\Psi^+\right\rangle_{at-ph} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_z \left|\sigma^+\right\rangle + |\uparrow\rangle_z \left|\sigma^-\right\rangle).$$

The two states $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$, defining the atomic qubit, correspond to the $|F=1, m_F = \pm 1\rangle$ Zeeman sublevels of the $5^2S_{1/2}, F=1$ hyperfine ground level. The purity of this state is limited only by the errors in preparation of the excited state[17], in our case we assume $e_{exc} = 0.5\%$ due to imperfections in the preparation of the initial state and resulting off-resonant excitation to different atomic states, leading to $V_{at-ph}^{(initial)} = 99.5\%$. The smaller visibility observed in the current experiments[11] is due to errors in the analysis of the atom-photon state which are described below. For the generation of atom-atom entanglement via entanglement swapping, the photon propagates via an optical fiber to a different location where the two-photon interference takes place. Recently we have demonstrated an optical fiber link of 300 m length[12], where the polarization errors were kept below 1% by active polarization control. Thus the remaining polarization errors in the fiber ($e_{pol} = 1\%$) reduce the visibility to

$$V_{at-ph} = (1-e_{exc})(1-e_{pol}) = 98.5\%.$$

This is the atom-photon visibility which is assumed before the photons enter the apparatus for the Bell-state measurement (BSM).

In the entanglement swapping process an additional error might occur due to mismatch in the two-photon interference which is assumed to be $e_{BSM} = 3\%$. The projection of the two atoms onto the entangled state is heralded by the coincidence detection (double click) of the two photons leaving two different output ports of the beamsplitter. Conditioned on this coincidence, the probability $p(|\Psi^-\rangle_{at-at})$ to get the desired entangled atom-atom state $|\Psi^-\rangle_{at-at}$ is

$$(1-e_{BSM})(\frac{1}{4} + \frac{3}{4}V_{at-ph}^2) + \frac{1}{4}e_{BSM} = 95.6\%, \qquad (4)$$

where the influence of the error e_{BSM} follows from (3).

Dark counts in the single photon detectors of the Bell-state analyzer will add spurious events. The fraction of wrong coincidence events is calculated as follows. The probability to get a photon from the first trapped atom at the beamsplitter is $\eta_1 = 1.3 \cdot 10^{-3} \times 0.6 = 0.78 \cdot 10^{-3}$, where the first number is the local efficiency for the generation of entangled atom-photon pairs (including the detection efficiency of single-photon detectors), while the second number accounts for the coupling and transmission losses in the fiber, as well as the limited time window for the coincidence detection. For the photon from the second atomic trap this number is higher due to the higher numerical aperture, $\eta_2 = 2.0 \cdot 10^{-3} \times 0.6 = 1.2 \cdot 10^{-3}$. Therefore the probability to detect a coincidence of the two photons is

$$p_{coincidence}^{(true)} = \frac{1}{4}\eta_1\eta_2 = 2.34 \cdot 10^{-7}. \qquad (5)$$

The factor $\frac{1}{4}$ accounts for the fact that only one out of four photonic Bell-states is detected. A “wrong” coincidence happens if one photon arrives at the beamsplitter and is detected in one detector while the other detector produces a dark count within the coincidence time window. For the detectors which will be used for this purpose (Perkin-Elmer SPCM-AQR15) the dark count rate is $r_{dc} \leq 50$ cps. For a coincidence time window of $\Delta T = 40$ ns the probability of such an event is

$$p_{coincidence}^{(dark)} \approx (\eta_1 + \eta_2) r_{dc} \Delta T = 3.96 \cdot 10^{-9}.$$

As the probability of detecting two dark counts as coincidence is negligible ($4 \cdot 10^{-12}$), the fraction of wrong events in the coincidence detection is $e_{dc} = 1.68\%$. Applying the relations (3) to the fidelity from (4) we obtain a resulting fidelity of $F_{at-at} = 94.4\%$ and visibility of $V_{at-at} = \frac{1}{3}(4F_{at-at} - 1) = 92.5\%$.

IV. EXPECTED VIOLATION OF BELL’S INEQUALITY

For the experimental test of the CHSH formulation of Bell’s inequality, the parameter S is measured, which is defined as

$$S := \left|\left\langle\sigma_\alpha\sigma_\beta\right\rangle + \left\langle\sigma_{\alpha'}\sigma_\beta\right\rangle\right| + \left|\left\langle\sigma_\alpha\sigma_{\beta'}\right\rangle - \left\langle\sigma_{\alpha'}\sigma_{\beta'}\right\rangle\right|. \qquad (6)$$

Here $\left\langle\sigma_\alpha\sigma_\beta\right\rangle$ is the expectation value of joint measurements on the spins of two particles where one spin is analyzed at an angle α and the other one at an angle β (we define these angles in terms of light polarization in the laboratory frame). According to Bell’s theorem, any theory with local hidden variables predicts $S \leq 2$. In quantum mechanics $S = 2\sqrt{2}$ is reached, e.g. for $\alpha = 0°$, $\alpha' = 45°$, $\beta = 22.5°$, $\beta' = -22.5°$.

In an experiment we measure the number of events "↑↑", "↓↓", "↑↓", "↓↑", where the "ups" and "downs" are the orientations of the spins with respect to the corresponding analysis directions α, β. We shall call these numbers $N_{\uparrow\uparrow}^{(\alpha,\beta)}$, $N_{\downarrow\downarrow}^{(\alpha,\beta)}$, $N_{\uparrow\downarrow}^{(\alpha,\beta)}$, $N_{\downarrow\uparrow}^{(\alpha,\beta)}$, while the total number of events per setting (α,β) is $N_s = N_{\uparrow\uparrow}^{(\alpha,\beta)} + N_{\downarrow\downarrow}^{(\alpha,\beta)} + N_{\uparrow\downarrow}^{(\alpha,\beta)} + N_{\downarrow\uparrow}^{(\alpha,\beta)}$. The expectation values are calculated as

$$\begin{aligned}\left\langle \sigma_\alpha \sigma_\beta \right\rangle &= \frac{1}{N_s}(N_{\uparrow\uparrow}^{(\alpha,\beta)} + N_{\downarrow\downarrow}^{(\alpha,\beta)} - N_{\uparrow\downarrow}^{(\alpha,\beta)} - N_{\downarrow\uparrow}^{(\alpha,\beta)}) \\ &= \frac{2}{N_s}(N_{\uparrow\uparrow}^{(\alpha,\beta)} + N_{\downarrow\downarrow}^{(\alpha,\beta)}) - 1. \end{aligned} \quad (7)$$

We note that

$$\begin{aligned} N_{\uparrow\uparrow}^{(\alpha,\beta)} &= N_s \cdot p_{\uparrow\uparrow}^{(\alpha,\beta)}, \\ N_{\downarrow\downarrow}^{(\alpha,\beta)} &= N_s \cdot p_{\downarrow\downarrow}^{(\alpha,\beta)}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} p_{\uparrow\uparrow}^{(\alpha,\beta)} &= p\left(|\uparrow\rangle_1^{(\alpha)} |\uparrow\rangle_2^{(\beta)} \right) \\ p_{\downarrow\downarrow}^{(\alpha,\beta)} &= p\left(|\downarrow\rangle_1^{(\alpha)} |\downarrow\rangle_2^{(\beta)} \right) \end{aligned}$$

are the probabilities for both particles to be measured in the state $|\uparrow\rangle$ ($|\downarrow\rangle$) along their respective analysis direction. For the atomic states the relations

$$\begin{aligned} |\uparrow\rangle^{(\alpha)} &= \cos(\beta-\alpha)\,|\uparrow\rangle^{(\beta)} + \sin(\beta-\alpha)\,|\downarrow\rangle^{(\beta)} \\ |\downarrow\rangle^{(\alpha)} &= \cos(\beta-\alpha)\,|\downarrow\rangle^{(\beta)} - \sin(\beta-\alpha)\,|\uparrow\rangle^{(\beta)} \end{aligned}$$

hold and therefore

$$\begin{aligned} \left|\Psi^-\right\rangle &= \frac{1}{\sqrt{2}}(|\downarrow\rangle_1^{(\alpha)} |\uparrow\rangle_2^{(\alpha)} - |\uparrow\rangle_1^{(\alpha)} |\downarrow\rangle_2^{(\alpha)}) \\ &= \frac{1}{\sqrt{2}}\Big(\cos(\beta-\alpha)\,|\uparrow\rangle_1^{(\alpha)} |\downarrow\rangle_2^{(\beta)} - \sin(\beta-\alpha)\,|\uparrow\rangle_1^{(\alpha)} |\uparrow\rangle_2^{(\beta)} \\ &\quad - \cos(\beta-\alpha)\,|\downarrow\rangle_1^{(\alpha)} |\uparrow\rangle_2^{(\beta)} - \sin(\beta-\alpha)\,|\downarrow\rangle_1^{(\alpha)} |\downarrow\rangle_2^{(\beta)} \Big). \end{aligned}$$

The probabilities $p_{\uparrow\uparrow}^{(\alpha,\beta)}$, $p_{\downarrow\downarrow}^{(\alpha,\beta)}$ are explicitely calculated in the following by applying the experimental detection probabilities and accuracies depending on the detection method.

A. Atomic state analysis via state-selective atom removal and fluorescence detection

When the entangled atom-atom state (2) with an initial visibility V_{at-at} is analyzed, we expect the probabilities

$$\begin{aligned} p_{\uparrow\uparrow}^{(\alpha,\beta)} &= p_{\downarrow\downarrow}^{(\alpha,\beta)} \\ &= \frac{1}{4}\left(1 - V_{at-at}(2a_{det}-1)^2 \cos(2(\beta-\alpha))\right). \end{aligned} \quad (9)$$

of detecting both particles in the state $|\uparrow\rangle$, respectively $|\downarrow\rangle$ along the directions (α,β). Inserting this into (6, 7, 8) we determine the expected parameter S

$$S^{(flr.)} = 2\sqrt{2}V_{at-at}(2a_{det}-1)^2. \quad (10)$$

For $V_{at-at} = 92.5\%$, $a_{det}^{(flr.)} = 95\%$ this gives $S^{(flr.)} = 2.12$, corresponding to an observable atom-atom visibility of $V^{(flr.)} = 74.9\%$.

B. Atomic state analysis via state-selective ionization

The limited detection efficiency for the ionization fragments leads to an asymmetry in the accuracy for the two measurement outcomes. The result where one of the channel electron multipliers registers a particle definitely means that an ionization has taken place (the probability of a dark count is low and therefore neglected). However, the result where no particle is registered contains also the events where the ionized fragments were not detected[18]. The probabilities in this case are

$$\begin{aligned} p_{\uparrow\uparrow}^{(\alpha,\beta)} &= \frac{1}{4}p_d^2\left(1 - V_{at-at}(2a_{ST}-1)^2\cos(2(\beta-\alpha))\right) \\ p_{\downarrow\downarrow}^{(\alpha,\beta)} &= \frac{1}{4}(2-p_d)^2 \times \\ &\quad \times \left(1 - \frac{p_d^2}{(2-p_d)^2}V_{at-at}(2a_{ST}-1)^2\cos(2(\beta-\alpha))\right), \end{aligned} \quad (11)$$

where we have set $p_d = p_{ionize} \cdot p_{det}$ for brevity. The parameter S is then given by

$$S^{(ioniz)} = 2\sqrt{2}V_{at-at}p_d^2(2a_{ST}-1)^2 - 2(1-p_d)^2. \quad (12)$$

This expression is exactly valid for $p_d \geq (1+\sqrt[4]{2})^{-1}$. For the parameters $V_{at-at} = 92.5\%$, $a_{ST} = 97.25\%$, $p_d = 95\%$ we get $S^{(ioniz)} = 2.10$.

V. STATISTICAL UNCERTAINTY FOR THE VIOLATION OF BELL'S INEQUALITY

In order to violate Bell's inequality the value of $S > 2$ has to be measured with sufficient statistical significance. Calling the standard deviation of the measured value ΔS, it has to be assured that

$$\frac{S-2}{\Delta S} \geq k, \quad (13)$$

where k is the number of standard deviations for the violation. Taking $k = 3$ gives a confidence level of $\geq 99.73\%$. The standard deviation ΔS depends on the number of measured events and shall be calculated in the following.

Using Gaussian error propagation we get from (7)

$$\Delta\left\langle \sigma_\alpha \sigma_\beta \right\rangle = \frac{2}{N_s}\sqrt{\Delta N_{\uparrow\uparrow}^2 + \Delta N_{\downarrow\downarrow}^2}.$$

The uncertainty of S is

$$\Delta S = \sqrt{\sum_{\alpha,\beta} \Delta\left\langle \sigma_\alpha \sigma_\beta \right\rangle^2}, \tag{14}$$

where $\alpha = 0°, 45°$, $\beta = 22.5°, -22.5°$.

Next, the statistical uncertainties of the event numbers have to be determined. Here we note that for a Bernoulli experiment the standard deviation of the expectation value is given by

$$\begin{aligned} \Delta N_{\uparrow\uparrow} &= \sqrt{N_{\uparrow\uparrow} p_{\uparrow\uparrow}(1-p_{\uparrow\uparrow})} = \sqrt{N_s}\sqrt{p_{\uparrow\uparrow}^2(1-p_{\uparrow\uparrow})}, \\ \Delta N_{\downarrow\downarrow} &= \sqrt{N_{\downarrow\downarrow} p_{\downarrow\downarrow}(1-p_{\downarrow\downarrow})} = \sqrt{N_s}\sqrt{p_{\downarrow\downarrow}^2(1-p_{\downarrow\downarrow})}. \end{aligned} \tag{15}$$

With these expressions the uncertainty of the S parameter is calculated for the two considered detection methods.

A. Fluorescence detection

Using the expression (9) and taking the specific angles for the Bell measurement we obtain

$$p_{\uparrow\uparrow}^{(\alpha,\beta)} = p_{\downarrow\downarrow}^{(\alpha,\beta)} = \frac{1}{4}(1 \mp \frac{1}{\sqrt{2}}V),$$

where $V = V_{at-at}(2a_{det}-1)^2$, the "-" sign is valid for the settings $(0°, \pm 22.5°)$, $(45°, 22.5°)$ while the "+" sign appears in the setting $(45°, -22.5°)$. This expression is inserted into (15) giving

$$\begin{aligned} &\Delta N_{\uparrow\uparrow} = \Delta N_{\downarrow\downarrow} \\ &\quad = \frac{\sqrt{N_s}}{4}\sqrt{(1 \mp \frac{1}{\sqrt{2}}V)^2(1-\frac{1}{4}(1 \mp \frac{1}{\sqrt{2}}V)),} \end{aligned}$$

Therefore for (α, β) equal to $(0°, \pm 22.5°)$ and $(45°, 22.5°)$

$$\Delta\left\langle \sigma_\alpha \sigma_\beta \right\rangle = \frac{1}{2\sqrt{2}\sqrt{N_s}}\sqrt{(1-\frac{1}{\sqrt{2}}V)^2(3+\frac{1}{\sqrt{2}}V)}$$

and for (α, β) equal to $(45°, -22.5°)$

$$\Delta\left\langle \sigma_\alpha \sigma_\beta \right\rangle = \frac{1}{2\sqrt{2}\sqrt{N_s}}\sqrt{(1+\frac{1}{\sqrt{2}}V)^2(3-\frac{1}{\sqrt{2}}V)}.$$

Using (14) we finally get

$$\begin{aligned} &\Delta S^{(flr.)} = \frac{1}{\sqrt{2}\sqrt{N}} \times \\ &\times \sqrt{3(1-\frac{1}{\sqrt{2}}V)^2(3+\frac{1}{\sqrt{2}}V) + (1+\frac{1}{\sqrt{2}}V)^2(3-\frac{1}{\sqrt{2}}V)}, \end{aligned} \tag{16}$$

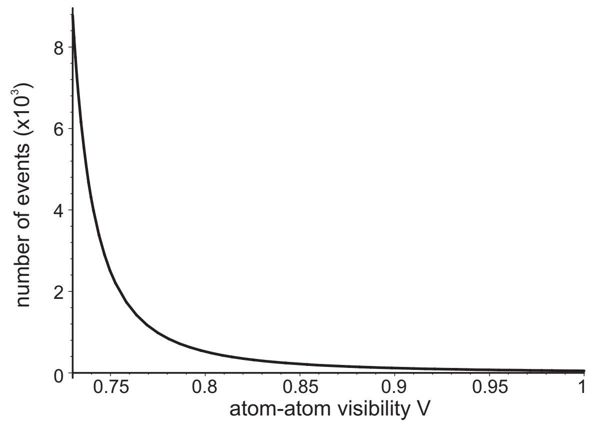

Figure 2: Number N of events necessary to violate Bell's inequality by 3 standard deviations using fluorescence detection as a function of the expected atom-atom visibility $V = V_{at-at}(2a_{det}-1)^2$.

where $N = 4N_s$ is the total number of events for all four settings together.

Inserting this result into the expression for violation of Bell's inequality (13) we can estimate the number of events necessary to achieve a certain confidence level. Figure 2 shows the dependence of the number of events N for a violation by 3 standard deviations as a function of the expected atom-atom visibility $V = V_{at-at}(2a_{det}-1)^2$. For a visibility of $V = 74.9\%$ we get $N = 2600$.

B. Ionization detection

Using the expression (11) and taking the specific angles for the Bell measurement we obtain

$$\begin{aligned} p_{\uparrow\uparrow}^{(\alpha,\beta)} &= \frac{1}{4}p_d^2\left(1 \mp \frac{1}{\sqrt{2}}V_{at-at}(2a_{ST}-1)^2\right), \\ p_{\downarrow\downarrow}^{(\alpha,\beta)} &= \frac{1}{4}(2-p_d)^2\left(1 \mp \frac{1}{\sqrt{2}}\frac{p_d^2}{(2-p_d)^2}V_{at-at}(2a_{ST}-1)^2\right), \end{aligned}$$

where $p_d = p_{ionize} \cdot p_{det}$. Again the "-" sign is for the settings $(0°, \pm 22.5°)$, $(45°, 22.5°)$ while the "+" sign appears in the setting $(45°, -22.5°)$. These are used for calculation of the uncertainty of the S parameter similar to the previous section. It is again inserted into (13) to estimate the necessary number of events. Figure 3 shows the dependence of the required number of events on the detection efficiency. Here we have assumed $V_{at-at}(2a_{ST}-1)^2 = 82.6\%$. For the detection efficiency $p_d = 95\%$ we get $N = 3470$ events.

VI. EXPERIMENTAL EVENT RATES AND MEASUREMENT TIME

In this section we estimate the repetition rate of the two-atom experiment and the overall measurement time necessary

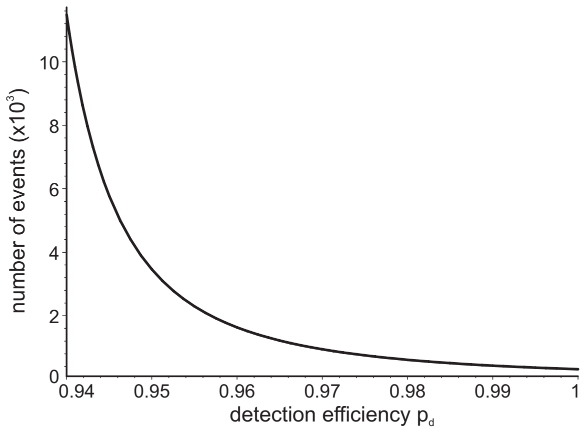

Figure 3: Number N of events necessary to violate Bell's inequality by 3 standard deviations with ionization detection as a function of the electron/ion detection efficiency p_d (including ionization probability). The assumed atom-atom visibility excluding the ionization detection efficiency is $V_{at-at}(2a_{ST}-1)^2 = 82.6\%$.

to violate Bell's inequality with sufficient statistical significance. In the current experiment, the sequence for generation of atom-photon entanglement consists of the preparation of the initial state by optical pumping ($\sim 5\,\mu$s) and excitation. Currently after every 20 preparation-excitation cycles the atom has to be cooled for $200\,\mu$s, which gives additional $10\,\mu$s per cycle. For the remote entanglement the emitted photon will be sent over an optical fiber of about 200 m length to the place where entanglement swapping is performed. Therefore a waiting time of $\frac{2\cdot 200\,\mathrm{m}}{\frac{2}{3}c} = 2\,\mu$s is necessary to send the photon and to receive a signal about the success or failure of the entanglement swapping procedure. This gives altogether $17\,\mu$s per cycle and a repetition rate of 58.8 kHz. Assuming a mean occupation number of each trap of 0.5 we get the duty cycle of the two-trap system of at least $(0.5)^2 = 0.25$. This results in an effective repetition rate of $0.25 \cdot 58.8\,\mathrm{kHz} = 14.7\,\mathrm{kHz}$. Together with the success probability (5) of the entanglement swapping process of $2.34 \cdot 10^{-7}$ we expect 1 atom-atom event in approximately 5 minutes. Depending on the detection method it is necessary to evaluate between 2600 and 3470 atom-atom events in order to violate Bell's inequality by 3 standard deviations. This requires a continuous measurement time between 9 and 12 days. By detection of a second Bell state during the BSM[19] this measurement time could be reduced by a factor of two.

VII. SUMMARY

We have shown the feasibility of a loophole-free test of Bell's inequality with entangled pairs of neutral atoms. By simultaneously exciting two single ^{87}Rb atoms in remote traps and detecting interference of the emitted photons it should be possible to entangle the atoms with a high fidelity. The two available methods of atomic state detection allow to violate Bell's inequality by achieving an $S \sim 2.1 > 2$ and to evaluate the complete ensemble of entangled atom pairs (i.e. without the need for a fair sampling assumption). Additionally, strict space-like separation of measurement events is obtainable by using a distance between the atomic traps of 300 m. Although very challenging, this approach is a promising candidate for a conclusive test of quantum mechanics against theories with local hidden variables.

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